

Provability of Functionnal Reactive Programming type system

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Motivation. It has been a decade that a Curry-Howard correspondance between Linear Temporal Logic (LTL) and Reactive Programming has been made (even if correspondance between LTL and other systems has been made before, by Davies [1] for instance). Reactive Programming is a paradigm of programmation where programs receive data over time and maintain properties over time. Linear Temporal Logic is a logic system where a new connective $\bigcirc$ allows us to describe the validity of formulas over time ($\bigcirc A$ means that the formula A will be true in the next time step). In addition to that, LTL helps to express the validity of a formula on at least one time-step in the future or on all time-step in the future with the two connectives $\Diamond$ and $\Box$ respectively. The system $LJ^\bigcirc$ from Kojima and Igarashi [2] does not deal with $\Box$ or $\Diamond$, whereas the type system for the Functionnal Reactive Programming FRP from Cave et al. [3] handles it by using fixpoint connectives μ and ν . In order to comply to some Reactive Programming properties such as causality, Temporal Logic has to reject formulas like $\bigcirc(A \vee B) \rightarrow \bigcirc A \vee \bigcirc B$ in the general case. The two systems $LJ^\bigcirc$ and FRP reject such a formula. One of our goal is to investigate provability of Cave et al. system using Kojima and Igarashi style of presentation. In fact, in Kojima and Igarashi system, time is handled by annotating the formulas of a given sequent with a natural number, which is more suitable for studying provability.

FRP $^\bigcirc$ and Kripke Semantic for FRP. As said before, Kojima and Igarashi system deals with temporality by annotating formulas of a sequent. We define a system of rules $FRP^\bigcirc$ in the Kojima and Igarashi style [2] together with provability equivalence regarding a μ and ν -free subsystem from FRP [3] and with a cut-elimination result. Here are the two equivalence results:

Proposition 1. *If $\Gamma^{\vec{m}} \vdash A^n$ is provable in $FRP^\bigcirc$, then for all $i \leq \min(n, \vec{m})$, $\vdash \bigcirc^{\vec{m}-i} \Gamma \vdash \bigcirc^{n-i} A$ is provable in FRP.*

Proposition 2. *If $\Theta; \Gamma \vdash A$ is provable in FRP, then for all i , $\Theta^{i+1}, \Gamma^i \vdash A^i$ is provable in $FRP^\bigcirc$.*

When looking at the two systems $LJ^\bigcirc$ and $FRP^\bigcirc$, we can easily notice that $LJ^\bigcirc$ proves more sequents than $FRP^\bigcirc$ and (to a certain extent) than FRP. In particular the formula $(\bigcirc A \rightarrow \bigcirc B) \rightarrow \bigcirc(A \rightarrow B)$ is provable in $LJ^\bigcirc$, but not in $FRP^\bigcirc$.

As Kojima and Igarashi proposed a Kripke semantic for $LJ^\bigcirc$, we propose a Kripke semantic $IM_{FRP}^\bigcirc$ by removing the condition of being an $IM^\bigcirc$ -frame in [2]. Satisfying all models will be harder for a given formula and it is precisely what we need for our soundness and completeness results relatively to $FRP^\bigcirc$:

Proposition 3. *For all Γ and A , the two following statements are equivalent:*

1. *For all $IM_{FRP}^\bigcirc (W, \leq, R, \Vdash)$, and for all $w \in W$, $w \Vdash \Gamma$ implies that $w \Vdash A$.*
2. *The sequent $\Gamma \vdash A$ is provable in $FRP^\bigcirc$.*

Future directions. The two systems from [2] and [3] are intuitionistic because intuitionistic systems ensure constructivity. Yet, since Griffin and Parigot's works [4] [5], it has been established that intuitionistic logic was not necessary to ensure constructivity. Another objective would be to consider a classical and constructive type system for FRP together with the fixpoint fragment from FRP (that we forgot with $FRP^\bigcirc$). The benefit of such consideration would be to have more elaborate primitives of programmation and to have a better control on the execution of the programs. However, a clear study of what can be proved in such type system has to be done, as the formula $\bigcirc(A \vee B) \rightarrow \bigcirc A \vee \bigcirc B$ seems to be provable in a classical version of LTL.

References

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